Spearman's Rank Correlation Coefficient and bike theft

Royal Geographical Society with IBG
Advancing geography and geographical learning

Specification links

AQA
A Level 3.4.2.4 Statistical skills *Inferential and relational statistical techniques to include Spearman's rank correlation and Chi-square test and the application of significance tests.*

Edexcel
A Level Appendix 1: Geographical skills. *This specification requires students to collect, analyse and interpret such information, and demonstrate the ability to understand and apply suitable analytical approaches for the different information types including, qualitative approaches such as coding and sampling and quantitative approaches such as measures of dispersion, measures of correlation and association from the following statistical tests: t-tests, Spearman's rank, Chi-squared, Gini Co-efficient, Lorenz curve.*

OCR
A Level Geographical Skills 4.4 Quantitative skills *b) tests of association and significance tests, such as Chi-squared, Spearman's rank, Mann-Whitney U test and T-test.*

Eduqas
A Level Appendix A Geographical Skills. *2. measures of correlation, including a scatter plot, lines of best fit and Spearman Rank.*

What is Spearman's Rank Correlation Coefficient?

Spearman's Rank is a statistical method used to test the strength of the relationship between two variables. It uses ranked data to test the relationship and calculates a fixed figure (between –1 and +1) to show the strength of that relationship:

- +1 indicates a perfect positive correlation
- -1 indicates a perfect negative correlation
- 0 shows there is no correlation at all between the two variables

Once the fixed figure has been calculated and a +1, -1 or 0 result is identified the numerical value must be tested statistically to see how significant the result is.

The test can be used for any two sets of data (variables) so long as it is raw data (or percentages or indices) which can be ranked. Below is the equation for Spearman's Rank Correlation Coefficient (with each part explained on the following page):

$$r_s = 1 - \frac{6\sum d^2}{n\left(n^2 - 1\right)}$$

r_s Spearman's Rank
$\sum$ the sum of
d^2 difference
n number of values

Test 1 a worked example. Spearman's Rank Correlation Coefficient: are areas with high levels of cycling more susceptible to bike theft?

This is a worked example to investigate whether cycling hotspots, such as the city of Oxford, have higher levels of bike theft. This statistical equation will calculate if there is a relationship (correlation) between the number of cyclists and the level of bike theft in an area.

Evidence at the national level shows that cycle theft is often higher in counties where more people cycle. This suggests that because there are greater opportunities for bike theft individuals suffer more from bike crime.

This resource uses bike theft data from www.data.police.uk, the government Walking and cycling statistics PDF 2019 report for England and specifically the CW0302: Proportion of adults that cycle, by frequency, purpose and local authority: England dataset from 2018-2019, based on the National Travel Survey and Active Lives Survey. Population data for each Police Force area was extracted from the 2020 ONS dataset Estimates of the population for the UK, England and Wales, Scotland and Northern Ireland.

In order to identify individuals who either commute to work or cycle regularly for leisure the *Total % of adults who cycle for at least 5 x p/w* has been selected for column 4 in Table 1 below.

January 2011 to December 2020 inclusive - Residential Population			
		Variable 1	Variable 2
Police Constabulary	Bike theft over a 10-year period	Bike theft per annum	The number of people who cycle x 5 times a week per annum
Metropolitan Police Service	149929	14993	394327
Greater Manchester Police	27065	2707	82234
West Yorkshire Police	17672	1767	48981
West Midlands Police	23385	2339	124614
Lancashire Constabulary	13577	1358	26835
South Yorkshire Police	9506	951	25362
Kent Police	12727	1273	37957
Hampshire Constabulary	27003	2700	42858
Thames Valley Police	41965	4197	85089
Northumbria Police	15012	1501	34166
Essex Police	17277	1728	31272
Avon and Somerset Constabulary	24313	2431	59254
Merseyside Police	13991	1399	35747
Sussex Police	18290	1829	41634
Police Service of Northern Ireland	6388	639	No data
South Wales Police	15344	1534	No data

Devon & Cornwall Police	6858	686	50271
Nottinghamshire Police	14033	1403	24846
West Mercia Police	8212	821	34756
Staffordshire Police	7630	763	12313
Derbyshire Constabulary	7577	758	20211
Hertfordshire Constabulary	11080	1108	26169
Cheshire Constabulary	9963	996	26494
Humberside Police	15709	1571	26666
Surrey Police	9370	937	49045
Cleveland Police	7275	937	3428
Leicestershire Police	14581	728	21184
Northamptonshire Police	6761	1458	16572
Cambridgeshire Constabulary	29233	676	62086
Norfolk Constabulary	9657	2923	38125
Dorset Police	9663	966	11733
Durham Constabulary	4043	966	7951
North Yorkshire Police	9563	404	14215
North Wales Police	3833	956	No data
Bedfordshire Police	7620	383	11481
Lincolnshire Police	8702	762	24359
Gwent Police	2999	870	No data
Suffolk Constabulary	8317	300	23601
Gloucestershire Constabulary	7177	832	2493318
Wiltshire Police	6089	718	10500
Warwickshire Police	5139	609	15026
Cumbria Constabulary	2380	514	14000
Dyfed-Powys Police	1275	238	
City of London Police	2994	128	944978

Table 1

This is a correlation exercise using the two variables. These two variables are two separate data sources: Bike theft (variable 1) and % of adults who cycle for at least 5 x p/w (variable 2). Ultimately you are investigating the relationship between these two variables i.e. is there a relationship between high levels of bike theft and areas with a high level of cycling. Spearman's Rank Correlation Coefficient statistically proves whether there is or is not a link (and how strong it is).

You will finish with a r_s (Spearman's Rank) value which will be used to identify whether correlation is:

- Nearer to 0, indicating the correlation is weak (either weak positive or weak negative).
- Close to ± 1, showing there is a strong positive correlation.
- Close to -1, revealing there is a strong negative correlation.

Finally, it is important to verify the result as meaningful. This proves the result was not calculated by chance and is revealed by a confidence level and a significance table.

Step 1
Before this statistical test is applied you formulate a **null hypothesis**. This is a theory which says there is no statistical relationship or significance between variables. This could be:

"There will be **no** significant relationship between bike theft and the number of people who cycle".

Step 2
It is first important to rearrange the data into a new format, to allow for ease of analysis (see Table 2 below).

The Police Service of Northern Ireland and the 4 Police Forces of Wales have been removed because the *Participation in walking and cycling* survey did not cover these regions.

Rank the data by filling out the (R^1) and (R^2) columns. Rank both sets of data from lowest to highest i.e., the **lowest** value gets rank **1**, the 2nd lowest gets rank 2 and so on.

To help you get started ranks 1, 2 and 3 for both variables have been filled in.

Number	Police Constabulary	Bike theft p/a	Rank (R^1)	The number of people who cycle 5 times a week p/a	Rank (R^2)	d $R^1 - R^2$	d^2
1	Metropolitan Police Service	14993		394327			
2	Greater Manchester Police	2707		82234			
3	West Yorkshire Police	1767		48981			
4	West Midlands Police	2339		124614			
5	Lancashire Constabulary	1358		26835			
6	South Yorkshire Police	951		25362			
7	Kent Police	1273		37957			
8	Hampshire Constabulary	2700		42858			
9	Thames Valley Police	4197		85089			
10	Northumbria Police	1501		34166			
11	Essex Police	1728		31272			
12	Avon and Somerset Constabulary	2431		59254			
13	Merseyside Police	1399		35747			
14	Sussex Police	1829		41634			
15	Devon & Cornwall Police	686		50271			
16	Nottinghamshire Police	1403		24846			

17	West Mercia Police	821		34756			
18	Staffordshire Police	763		12313			
19	Derbyshire Constabulary	758		20211			
20	Hertfordshire Constabulary	1108		26169			
21	Cheshire Constabulary	996		26494			
22	Humberside Police	1571		26666			
23	Surrey Police	937		49045			
24	Cleveland Police	937		3428	1		
25	Leicestershire Police	728		21184			
26	Northamptonshire Police	1458		16572			
27	Cambridgeshire Constabulary	676		62086			
28	Norfolk Constabulary	2923		38125			
29	Dorset Police	966		11733			
30	Durham Constabulary	966		7951	2		
31	North Yorkshire Police	404		14215			
32	Bedfordshire Police	383	3	11481			
33	Lincolnshire Police	762		24359			
34	Suffolk Constabulary	300	2	23601			
35	Gloucestershire Constabulary	832		2493318			
36	Wiltshire Police	718		10500	3		
37	Warwickshire Police	609		15026			
38	Cumbria Constabulary	514		14000			
39	City of London Police	128	1	944978			
						∑	

Table 2

If you have two numbers that are the same you cannot give them the same whole number, neither can you rank one over the other — so you simply find the average of the two ranks. Be careful to "jump" to the next rank correctly. Mrs Spicer gives an excellent explanation in this video (from 3 minutes in) on how to do this.

Step 1
Fill out column 7 of Table 2 by working out R^1 - R^2 (subtracting each piece of R^1 data from R^2) to calculate *d* i.e., the difference.

Step 2
For column 8 square the data from step 1.

Step 3
Sum up the final column and fill in the ∑ value in the bottom right cell. Go back to the equation. This figure is your $\sum d^2$ part of the equation. The n value is the number of values, in this case, there are 39.

At this stage it is advisable to rewrite out the Spearman's Rank Correlation Coefficient equation and populate it these figures. When you rewrite the final equation (or type it into a calculator) remember to start with 1- before the fraction.

Step 4
Is your r_s your value a positive or negative result? How strong is the correlation: is it close to ± 1 or -1? On page 4 in A Guide to Spearman's Rank by the Royal Geographical Society (with IBG) there is a useful spectrum to visualise the continuum from ± 1 to -1.

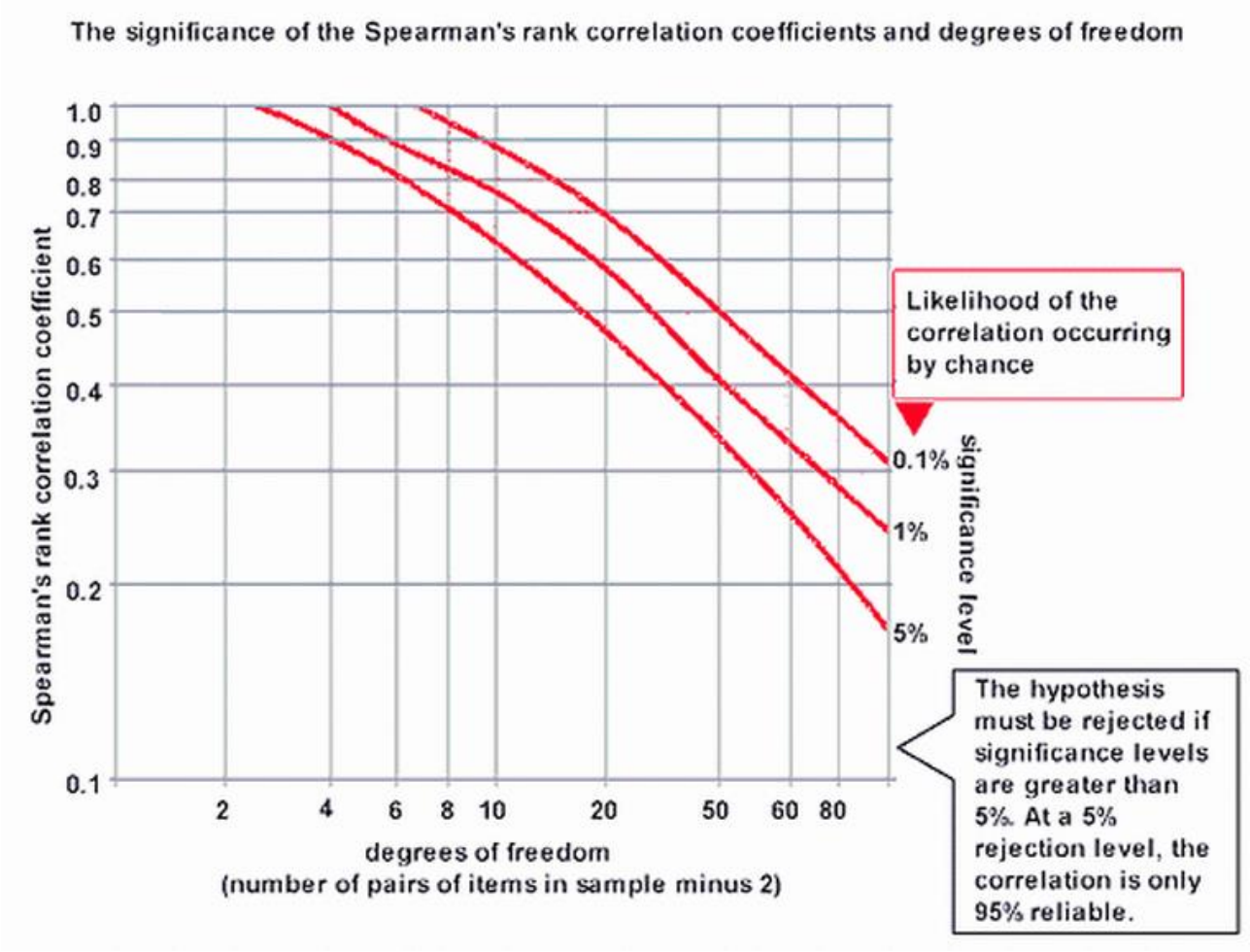
Step 5
How likely is it that this result was calculated by chance? To work this final bit out you need to consult a significance table (Table 3 below). Normally, you look for the column shown as 95% confidence (or sometimes listed in reverse as 0.5 level or 5% (i.e., there is a 5% likelihood that this result was calculated by chance).

If your result comes out as higher than the number shown for the number of data sets used (39) then you can be confident the result is not down to random chance. 95% is a high rate of confidence, and it is generally considered good enough.

Number of pairs of data (n)	0.10 10% chance 90% confident	0.05 5% chance 95% confident	0.01 1% chance 99% confident
5	0.9	1	1
6	0.829	0.886	1
7	0.714	0.786	0.929
8	0.643	0.738	0.881
9	0.6	0.700	0.833
10	0.564	0.648	0.794
12	0.503	0.587	0.727
14	0.464	0.538	0.679
16	0.429	0.503	0.635
18	0.401	0.472	0.600
20	0.380	0.447	0.570
22	0.361	0.425	0.544
24	0.344	0.406	0.521
26	0.331	0.390	0.501
28	0.317	0.375	0.483
30	0.306	0.362	0.467
31	0.301	0.356	0.459
32	0.296	0.350	0.452
33	0.291	0.345	0.446
34	0.287	0.340	0.439
35	0.283	0.335	0.433
36	0.279	0.330	0.427
37	0.275	0.325	0.421
38	0.271	0.321	0.415
39	0.267	0.317	0.410

Table 3 http://webspace.ship.edu/pgmarr/geo441/tables/spearman%20ranked%20correlation%20table.pdf

Step 6
Sometimes you may be presented with a graph to calculate the significance of a result. It is important that you can work this out from both a table (such as Table 3) *and* by using a degrees of freedom graph, like the one below (Graph 1).



Graph 1 A degrees of freedom graph

Test 2 Spearman's Rank: is there correlation between high rates of bike theft and areas with high levels of cycling for leisure?

The CSEW offer a breakdown of the data set *Total % of adults who cycle for at least 5 x p/w* (used in Test 1) into those that cycle for work, and those that cycle for leisure.

This is a second Spearman's Rank Correlation Coefficient task. Return to the CW0302: Proportion of adults that cycle, by frequency, purpose and local authority: England dataset from 2018-2019 this time to complete a Spearman's Rank Correlation Coefficient statistical test using the data column *Cycling for leisure five times per week*.

Is correlation between high rates of bike theft and areas with high levels of cycling for leisure? Use Table 4 on the next page to calculate the correlation. Repeat and work through the same steps 1-6 as outlined in Test 1.

Number	Police Constabulary	Bike theft p/a	Rank (R^1)	The number of people who cycle **for Leisure** 5 times a week p/a	Rank (R^2)	d $R^1 - R^2$	d^2
1	Metropolitan Police Service	14993		71695			
2	Greater Manchester Police	2707		34028			
3	West Yorkshire Police	1767		23324			
4	West Midlands Police	2339		53406			
5	Lancashire Constabulary	1358		19516			
6	South Yorkshire Police	951		7045			
7	Kent Police	1273		11070			
8	Hampshire Constabulary	2700		15207			
9	Thames Valley Police	4197		24655			
10	Northumbria Police	1501		9725			
11	Essex Police	1728		11913			
12	Avon and Somerset Constabulary	2431		11954			
13	Merseyside Police	1399		8579			
14	Sussex Police	1829		13961			
15	Devon & Cornwall Police	686		25343			
16	Nottinghamshire Police	1403		9110			
17	West Mercia Police	821		15427			
18	Staffordshire Police	763		4397			
19	Derbyshire Constabulary	758		7432			
20	Hertfordshire Constabulary	1108		10705			
21	Cheshire Constabulary	996		1592			
22	Humberside Police	1571		8892			
23	Surrey Police	937		16747			
24	Cleveland Police	937		1234			
25	Leicestershire Police	728		6355			
26	Northamptonshire Police	1458		6779			

27	Cambridgeshire Constabulary	676		9149			
28	Norfolk Constabulary	2923		15431			
29	Dorset Police	966		4920			
30	Durham Constabulary	966		3710			
31	North Yorkshire Police	404		6798			
32	Bedfordshire Police	383		2908			
33	Lincolnshire Police	762		8373			
34	Suffolk Constabulary	300		8374			
35	Gloucestershire Constabulary	832		6768			
36	Wiltshire Police	718		7500			
37	Warwickshire Police	609		4623			
38	Cumbria Constabulary	514		7500			
39	City of London Police	128		9			
						$\sum$	

Table 7

Further work

- Mrs Spicer on Spearman's Rank Correlation Coefficient, measuring correlation between earthquakes and the number of deaths https://www.youtube.com/watch?v=5PnyKjxWXDA
- A guide to Spearman's Rank from the Royal Geographical Society (with IBG) https://www.rgs.org/CMSPages/GetFile.aspx?nodeguid=882169d2-8f96-4c55-84f5-fbb7614870e9&lang=en-GB
- Best Song Ever https://www.stem.org.uk/resources/elibrary/resource/36017/best-song-ever
- How safe is your bike? https://www.met.police.uk/cp/crime-prevention/theft-of-a-bike/how-safe-is-your-bike/

Answers

1. Below is the complete calculation for Spearman's Rank Correlation Coefficient: are areas with high levels of cycling more susceptible to bike theft?

Number	Police Constabulary	Bike theft p/a	Rank (R^1)	The number of people who cycle 5 times a week p/a	Rank (R^2)	d $R^1 - R^2$	d^2
1	Metropolitan Police Service	14993	39	394327	37	2	4

2	Greater Manchester Police	2707	36	82234	34	2	4
3	West Yorkshire Police	1767	31	48981	29	2	4
4	West Midlands Police	2339	33	124614	36	-3	9
5	Lancashire Constabulary	1358	24	26835	20	4	16
6	South Yorkshire Police	951	18	25362	16	2	4
7	Kent Police	1273	23	37957	25	-2	4
8	Hampshire Constabulary	2700	35	42858	28	7	49
9	Thames Valley Police	4197	38	85089	35	3	9
10	Northumbria Police	1501	28	34166	22	6	36
11	Essex Police	1728	30	31272	21	9	81
12	Avon and Somerset Constabulary	2431	34	59254	32	2	4
13	Merseyside Police	1399	25	35747	24	1	1
14	Sussex Police	1829	32	41634	27	5	25
15	Devon & Cornwall Police	686	8	50271	31	-23	529
16	Nottinghamshire Police	1403	26	24846	15	11	121
17	West Mercia Police	821	14	34756	23	-9	81
18	Staffordshire Police	763	13	12313	6	7	49
19	Derbyshire Constabulary	758	11	20211	11	0	0
20	Hertfordshire Constabulary	1108	22	26169	17	5	25
21	Cheshire Constabulary	996	21	26494	18	3	9
22	Humberside Police	1571	29	26666	19	10	100
23	Surrey Police	937	16.5	49045	30	-13.5	182.25
24	Cleveland Police	937	16.5	3428	1	15.5	240.25
25	Leicestershire Police	728	10	21184	12	-2	4
26	Northamptonshire Police	1458	27	16572	10	17	289
27	Cambridgeshire Constabulary	676	7	62086	33	-26	676
28	Norfolk Constabulary	2923	37	38125	26	11	121
29	Dorset Police	966	19.5	11733	5	14.5	210.25
30	Durham Constabulary	966	19.5	7951	2	17.5	306.25
31	North Yorkshire Police	404	4	14215	8	-4	16
32	Bedfordshire Police	383	3	11481	4	-1	1

33	Lincolnshire Police	762	12	24359	14	-2	4
34	Suffolk Constabulary	300	2	23601	13	-11	121
35	Gloucestershire Constabulary	832	15	2493318	39	-24	576
36	Wiltshire Police	718	9	10500	3	6	36
37	Warwickshire Police	609	6	15026	9	-3	9
38	Cumbria Constabulary	514	5	14000	7	-2	4
39	City of London Police	128	1	944978	38	-37	1369
						∑	5329

r_s (Spearman's Rank Correlation Coefficient) final equation is 31974 ÷ 57798 = 0.5532025329596. This is a positive result showing that there *is* correlation between the amount people cycle and how often their bikes are stolen.

Using the significance table, we can say this result did not occur by chance with 99% confidence.

2. Below is the second complete calculation for Spearman's Rank Correlation Coefficient: is there correlation between high rates of bike theft and areas with high levels of cycling for leisure?

Number	Police Constabulary	Bike theft p/a	Rank (R^1)	The number of people who cycle **for Leisure** 5 times a week p/a	Rank (R^2)	*d* $R^1 - R^2$	d^2
1	Metropolitan Police Service	14993	39	71695	39	0	0
2	Greater Manchester Police	2707	36	34028	37	-1	1
3	West Yorkshire Police	1767	31	23324	34	-3	9
4	West Midlands Police	2339	33	53406	38	-5	25
5	Lancashire Constabulary	1358	24	19516	33	-9	81
6	South Yorkshire Police	951	18	7045	13	5	25
7	Kent Police	1273	23	11070	25	-2	4
8	Hampshire Constabulary	2700	35	15207	29	6	36
9	Thames Valley Police	4197	38	24655	35	3	9
10	Northumbria Police	1501	28	9725	23	5	25
11	Essex Police	1728	30	11913	26	4	16

12	Avon and Somerset Constabulary	2431	34	11954	27	7	49
13	Merseyside Police	1399	25	8579	19	6	36
14	Sussex Police	1829	32	13961	28	4	16
15	Devon & Cornwall Police	686	8	25343	36	-28	784
16	Nottinghamshire Police	1403	26	9110	21	5	25
17	West Mercia Police	821	14	15427	30	-16	256
18	Staffordshire Police	763	13	4397	6	7	49
19	Derbyshire Constabulary	758	11	7432	14	-3	9
20	Hertfordshire Constabulary	1108	22	10705	24	-2	4
21	Cheshire Constabulary	996	21	1592	3	18	324
22	Humberside Police	1571	29	8892	20	9	81
23	Surrey Police	937	16.5	16747	32	-15.5	240.25
24	Cleveland Police	937	16.5	1234	2	14.5	210.25
25	Leicestershire Police	728	10	6355	9	1	1
26	Northamptonshire Police	1458	27	6779	11	16	256
27	Cambridgeshire Constabulary	676	7	9149	22	-15	225
28	Norfolk Constabulary	2923	37	15431	31	6	36
29	Dorset Police	966	19.5	4920	8	11.5	132.25
30	Durham Constabulary	966	19.5	3710	5	14.5	210.25
31	North Yorkshire Police	404	4	6798	12	-8	64
32	Bedfordshire Police	383	3	2908	4	-1	1
33	Lincolnshire Police	762	12	8373	17	-5	25
34	Suffolk Constabulary	300	2	8374	18	-16	256
35	Gloucestershire Constabulary	832	15	6768	10	5	25
36	Wiltshire Police	718	9	7500	15.5	-6.5	42.25
37	Warwickshire Police	609	6	4623	7	-1	1
38	Cumbria Constabulary	514	5	7500	15.5	-10.5	110.25
39	City of London Police	128	1	9	1	0	0
						∑	3699.5

r_s (Spearman's Rank Correlation Coefficient) final equation for this second test is 3699.5 ÷ 57798 = 0.0640074051005. This proves there *is* positive correlation between areas that have high levels of

cycling **for leisure** and high level of bike theft — but it is not a strong relationship between the two data sets.

This result is interesting because we can deduce that the result from test 1 (for all cycling, 5 times a week) might have a stronger positive correlation due to cyclists who are commuters (i.e., not cycling for leisure). In order to know for certain you will need to do a third Spearman's Rank Correlation Coefficient test on *Cycling for travel 5 times a week*, again using the CW0302: Proportion of adults that cycle, by frequency, purpose and local authority: England dataset from 2018-2019.

However, after using the significance table, we cannot be confident in this result as the value fails to meet the 90% confidence threshold (0.267).